Asset Allocation for Phased Withdrawal Plans

Asset Management Seminar

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Abstract: The paper compares life annuities to four self-annuitization strategies of phased withdrawal. While each of the strategies has its advantages and drawbacks, the risk-value approach is taken to compare different withdrawal plans. As an indicator of returns, I compare the periodic payments, as well as the expected bequest, while as risk I take the probability of consumption shortfall. Due to the stochastic nature of asset returns, most numbers are calculated using Monte-Carlo simulation for obtaining different estimates of risk and return. The results show that there is no dominant strategy among these in terms of the risk and returns. Each strategy offers a specific risk-return combination, and depending on preferences of a retired person, the desirable strategy will change.

Keywords: Asset management; pension portfolio; life annuities; DAX; withdrawal plans; Monte-Carlo; expected shortfall; bequest; periodic payments, shortfall probability, mortality rates, survival probability.

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1. Introduction

The primary objective of this paper is to replicate "*Betting on death and capital markets in retirement: a shortfall risk analysis of life annuities versus phased withdrawal plans",* a study conducted by Dus, Maurer, and Mitchell (2005), with updated data. In my work I have used largely the same data, methodology, and research design as in the original paper. However, there are several deviations from the paper as well, about which I will speak below. In the next section I will briefly describe the methodology of my calculations. For a more detailed explanation of theory, methodology, and calculations one can refer to the original paper.

The paper compares life annuities to four self-annuitization strategies of phased withdrawal. A life annuity is a contract between the issuer, usually an insurance company, and the purchaser promising to pay a certain amount, based on the initial premium, at fixed intervals for as long as the purchaser is alive. Life annuities have several advantages that make them appealing for the retirees. First, they are relatively simple and easy to understand for regular people. Second, they almost eliminate uncertainty and are less risky. One way in which they are more stable is that the purchaser is guaranteed to receive the payments till the end of his life, and there is no risk of running out of funds and receiving nothing at later stages of his life. Additionally, as the payments are not connected to any asset classes or economic conditions, the amount they receive is constant. There are several types of life annuities, but the one used in this paper is a real immediate annuity, i.e. it is adjusted for inflation, which further adds stability, and starts immediately after the purchase.

Life annuities, however, have also several drawbacks. First, they do not provide liquidity and flexibility to the purchaser. Once the initial premium is paid to the insurance company, it cannot be withdrawn at a later date. This may repel elderly people, since they expect to have some, usually medical, unforeseen costs. Another disadvantage is that life annuities do not leave any bequest to the heirs of the purchaser, even if the purchaser passes away early. Thus, there is a risk of "not using" the invested money in scenarios of early death. Elderly people concerned about leaving a bequest will not choose life annuities as a retirement strategy. Finally, as life annuities usually offer some guarantees to stay competitive, they may have high administrative costs compared to other retirement strategies.

As opposed to the life annuities, the phased withdrawal plans distribute the initial capital among different asset classes, like bonds and equity. These plans are the opposite of the life annuities, in the sense that they do not have the risks associated with annuities, but involve uncertainty. First, phased withdrawal plans provide liquidity to the investors, as they can pull out the remaining capital at any point if they need to. And as they have the full control over their capital, the remaining amount of capital remains on their account in the form of a bequest in case of early death. Conversely, some of these plans introduce longevity risk, i.e. the risk of living longer than initially expected, thus exhausting the invested capital and living the last years, when retirees need the most money, in poverty. In addition, the stochastic nature of underlying asset returns means that the asset values as well as periodic payment amounts will also vary in time. Although this can have both positive and negative consequences for the retiree, it creates undesirable uncertainty.

To compare the strategies, we need a quantitative measure of returns and risks. As the exact risk taking of the retirees is unknown, it will be hard to compare them by maximizing a utility function. Consequently, I will follow the original paper and use a risk-value approach, i.e. I will assume that individuals prefer more returns and less risk, and will try to maximize the returns and minimize the risk. As returns we can compare the periodic payments, as well as the expected bequest, while as risk we take the probability of consumption shortfall, that is the probability of not reaching a certain benchmark of periodic payments. As the benchmark I take the fixed real annual payments of life annuities. Additionally, the severity of the shortfall will also be considered as a measure of risk.

2. Calculations

Determining Annuity Benefits

For calculating the life annuity payments, I use the DAV2004R annuitant mortality table for Germany. From this table I take the data for conditional mortality rates of German annuitants in 1999, which is the base year of this table, for both male and female. This mortality table also provides values for the exponential trend function. As the life expectancy has increased since 1999, for example due to improvements in healthcare, better living conditions, and medical innovations, we should project the 1999 year mortalities to the present day. From these values we, first, calculate the mortality rate reduction factor for each year. For that I use the following formula:

Reduction factor = exp [-F(X)] * (Projection Year - 1999)]

where F(X) is the trend function value at a given year. These reduction factors are then multiplied by the respective mortality rates to get the amended (up to date) mortality rates for each year. Thereafter, the trend adjusted survival probabilities are calculated and discounted. The next step is calculating the annuity factor, which is the sum of all the trend adjusted survival probabilities. After accounting the annuity factor for the expense loading, and dividing the initial premium by it, I get the immediate annual life-long real annuity benefit.

Following Albrecht and Maurer (2002), as well as Dus, Maurer, and Mitchell (2005), I make the same assumptions about the parameters of these calculations. The total expense loading relative to the pure actuarial premium is 2.785%, the real interest rate for discounting is 1.5%, and the initial premium is 100 €. I calculate the annuity payments for male and female, both aged 65. For male the annual benefit is 5.1583, and for female is 4.4896 p.a. As female are expected to live longer than male, and given the way the yearly payments are calculated, it is normal that the yearly amount they receive for the same premium is lower than for men.

It is also important to mention that the table used for my calculations goes till the age of 121, whereas in the original paper it goes till 110. Consequently, for all my further calculations the maximum possible duration of any strategy is 11 years more than in the original.

Stochastic returns

As alternative options to the immediate real life annuity payments, I consider four phased withdrawal strategies. In these strategies the retiree invests his/her retirement money into different assets, in our case into a mutual fund consisting of bonds and stocks. His/her investment earns some yearly stochastic return, and at the same time the retiree withdraws a certain amount each year for consumption purposes. However, before calculating the risk and return patterns for the four strategies and comparing them to the annuity, I will first show the calculations of the interest rates.

Following Dus, Maurer, and Mitchell (2005), I first make the assumption that the invested asset values follow a geometric random walk with drift. From this assumption it follows that the yearly log-returns are i.i.d. and are normally distributed with some mean and variance. For estimating these parameters, I will use historical data for DAX and REXP for the period from 1967 to 2018, which is all currently available data. These indices are proxies for German blue-chip stocks, i.e. equity, and constant maturity German government bonds respectively. Additionally, I will use data from German CPI for the same time period for calculating inflation. Again following the original authors, the portfolio is annually rebalanced and equally split between bonds and equity. One deviation I make here from the original paper is **not** taking into account the administrative costs of 0.5% p.a.

From this data I got real log average return for bonds and equity to be 0.0334 and 0.0335 respectively, while their standard deviations are 0.0445 and 0.2316, and correlation coefficient is 0.0602. While it is a little puzzling that stocks and bonds returns are almost identical, stocks are much more volatile, as expected. Given a fifty-fifty portfolio, the portfolio returns and standard deviation are 0.0335 and 0.1192 respectively (the numbers are rounded by MatLab). Taking these parameters as inputs, I use Monte-Carlo simulation to generate a number of evolution paths for the returns, which in turn generate evolution paths for risk and return measures of four withdrawal plans. For my calculations I take the number of paths as 1 000, but it can be easily changed to higher numbers if necessary.

Finally, I transform the simulated log returns into gross returns for each path and period, and then average them over the number of paths to get the gross return per period, which is used in calculations for withdrawal plans. Additionally, the expected gross return over the future period is $E(1 + R_t) = \exp[0.0335 + 0.5 \cdot 0.1192^2] = 1.0414$.

Withdrawal Strategies

Now that I have the required returns, I can proceed to calculating the risk and return measures for all the withdrawal plans. In this section I will briefly introduce the plans and the ways to calculate the required values. To make it comparable to the real annuity, the initial amount invested in either of these plans is also equal to 100 €.

1. Withdrawal Plans with Fixed Benefits

Under this withdrawal plan the retiree sells a fixed amount from his asset value for consumption. The fixed amount is set to be equal to the annuity payment, i.e. 5.1583 (for male). Thus, this strategy effectively replicates the life annuity in terms of yearly payments, but it does not have the limitations of it, i.e. lack of control and no potential bequest. Nevertheless, the yearly payments, despite being a fixed amount, depend on the stochastic returns, which may potentially create situations when the remaining asset value is less than the fixed payment amount, or is zero during the life of the person. Formally, the yearly payment

B_t and the remaining asset value V_t are given in the following way, where (1+R) is the gross return calculated from the simulated returns.

$$B_t = \min(B, V_t) \qquad V_{t+1} = (V_t - B_t)\cdot(1+R_{t+1}) = \begin{cases}(V_t - B)(1+R_{t+1}) & V_t > B \\ 0 & V_t \le B\end{cases}.$$

2. Fixed Percentage Rule

This, and the next two withdrawal rules have variable benefits. The general setup for these three plans is the following, where omega is the fraction of current wealth withdrawn that period.

$$B_t = \omega_t \cdot V_t \qquad V_{t+1} = (V_t - B_t)\cdot(1+R_{t+1}) = (1-\omega_t)\cdot V_t \cdot (1+R_{t+1})$$

As in the case of the first strategy, both the yearly payments and the remaining asset values are random variables, since they depend on the stochastic gross returns.

The fixed percentage rule is the simplest of the three, since the specified withrdrawal fraction is fixed over time. In the case of this paper I take it as 5.1583%, which is the percent that will make the first payment equal to the annuity payment.

3. 1/T rule

In this withdrawal rule the specified fraction is no longer constant, and is equal to 1/(T-t), where T is the maximum possible duration of the plan and t is the time passed after retirement. As the mortality table goes till 121 years, and the age of retirement is 65, the maximum possible duration of the plan is taken as 57, and people are dead with the probability of 1 after it. As (T-t) will decrease as the person becomes older, the fraction will increase over time.

4. 1/E[T(x)] Rule

Under this withdrawal rule the benefit/remaining wealth fraction is determined in the following way:

$$\frac{B_t}{V_t} = \omega_t = \frac{1}{\mathrm{E}[T(x+t)]}$$

E[T(x)] is the expected remaining lifetime of an individual at age x (x is 65 in our case) at different periods of time. Similar to the previous rule, the expected remaining lifetime declines with age, thus the fraction increases over time. Expected remaining lifetime at age (x+t) is calculated by summing up the survival probabilities of a person from (x+t) till T.

Risk and return measures

The approach taken in the original paper (Dus, Maurer, and Mitchell (2005)) is to develop analytical closed form solution for probability distribution of future risk and return patterns for the three variable benefit rules. It is possible to do given our initial assumption that log returns are normally distributed. However, the same technique cannot be used for the fixed benefit rule. It is the only strategy where the asset value may hit zero during the life of the retiree, thus making its probability distribution of future values unknown. The paper suggests using Monte-Carlo simulation for this withdrawal rule. Here I make another deviation from the original paper, since I do not use closed form solutions for the variable benefit plans, and use Monte-Carlo simulation for all four strategies.

As already mentioned above, one of the ways to quantify the riskiness of a certain strategy is the Shortfall Probability, i.e. the probability that the periodic payment at a certain point in time is less than the target (annuity payment). Dus, Maurer, and Mitchell (2005) suggest also using Shortfall Expectation to incorporate not only the probability of a shortfall, but its severity as well. They define it as $SE(Bt) = E\,[max\,(z - Bt,0)] = MEL(Bt) \cdot SP(Bt)$, where MEL stands for mean excess loss, and is calculated as $MEL(Bt) = E\,[\,z - Bt \mid Bt < z]$. Thus, shortfall expectation is the average loss per period weighted by its probability. It is also useful to calculate the expected present value of the shortfall, defined as:

$$EPVShortfall = \sum_{t=0}^{l-x} \frac{{}_t p_x SE(\mathrm{B_t})}{(1+R_f)^t}$$

where R_f is the risk free rate used for discounting. As the risk free rate I have taken 1.29%, which is the number given by KPMG 2018 Cost of Capital update for Germany. This value is used for comparing different strategies, as well as for optimization, where I minimize it as a measure of risk.

As measures of returns the paper uses expected present value of benefits and the expected present value of bequest in case of death. These are defined as:

$$EPVBenefits = \sum_{t=0}^{l-x} \frac{{}_t p_x E(\mathrm{B_t})}{(1+R_f)^t}, \text{ and}$$

$$EPVBequest = \sum_{t=1}^{l-x} \frac{{}_{t-1} p_x q_{x+t} E(\mathrm{V_t})}{(1+R_f)^t}$$

It is worth mentioning that both EPV of Shortfall and Bequests are zero for annuity payments, as it has no bequest potential and has a steady flow of fixed payments which are equal to the benchmark (itself), i.e. has no shortfall probability.

To optimize the plans, I minimize the EPVShortfall with regard to asset allocation. The way I proceeded in my calculations was changing the weights of bonds and equity by the increments of 5%, i.e. 0% bonds, 100%equity, then 5% bonds, 95% equity, etc. This is another slight deviation from the paper, as Dus, Maurer, and Mitchell (2005) use 2 additional factors for optimization, besides asset allocation weights.

3. Results

In this section I will show my results, interpret them, and compare to the result in the original paper. I will try to explain any major differences in the results. The plots shown here are all done for a 65-year-old male retiree with an evenly split asset allocation. Each line in the plots is numbered in the same order as discussed above, e.g. B3per corresponds to the 1/T rule strategy. On the x-axis are the years passed since retirement, i.e. the further right, the higher the age of the retiree.

The first figure shown plots yearly benefits paid by each of the phased withdrawal strategies. The benefits are presented as a percentage of periodic life annuity payment to make the comparison easier. The fixed benefit rule stays at 100% for approximately the first 20 years, as the payments are chosen to be the same as annuity payments. However, we can see that they start declining afterwards due to the increasing probability of shortfall associated with stochastic returns of the underlying assets. The fixed fraction rule also starts at 100%, as the fraction was chosen in that way. Yet, it too starts declining afterwards, although at a slower rate than the fixed payment strategy. This also contradicts the results in the original paper, where the fixed fraction rule benefits are increasing over time. The difference is explained by the fact that in the original paper the periodic withdrawal fraction is less than the gross interest rate, i.e. more money is added than withdrawn each year. In my case it is the opposite, as the interest rate is lower than the withdrawal fraction, 5.1583% > 4.14% and 1.0414 * (1-0.051583) = 0.987 < 1, which means it decreases each period.

For 1/T the benefits start at a low level of approximately 40% of annuity payment. However, they exponentially increase with age, surpassing the annuity level at older ages (>95). The reason for this is that low withdrawal rates at early stages allow the assets to accumulate over time, and at later stages not only do the assets have higher value, but the fraction increases as well. They sharply fall at the last year by construction. The benefits paid by the 1/E(T) rule start at approximately 85%, then steadily increase till its maximum point at 130% after 18 years, and start a steep decline to 0 after 33 years. Such behavior comes from the increasing withdrawal factor over time. Unlike the 1/T rule, it both increases faster, and starts at a higher rate, which does not allow the assets to accumulate. The behavior of all the plots, excluding the fixed fraction rule, is almost exactly the same as in the original paper.

Figure 1: Mean Benefit of Withdrawal Plan Conditional on Survival, 50-50: Life Annuity Benchmark

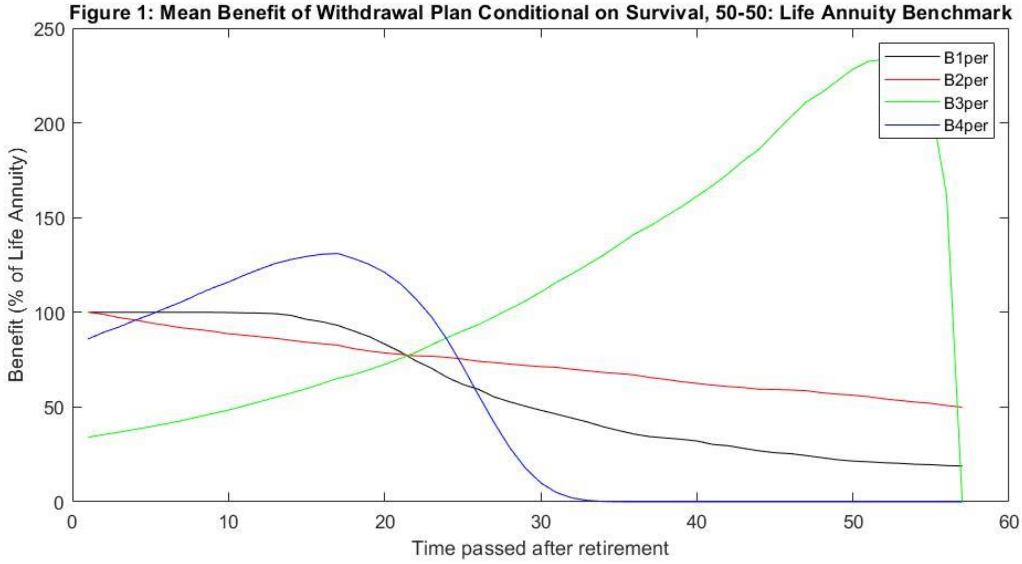

Figure 2: Shortfall Probability of Withdrawal Plan Conditional on Survival, 50-50: Life Annuity Benchmark

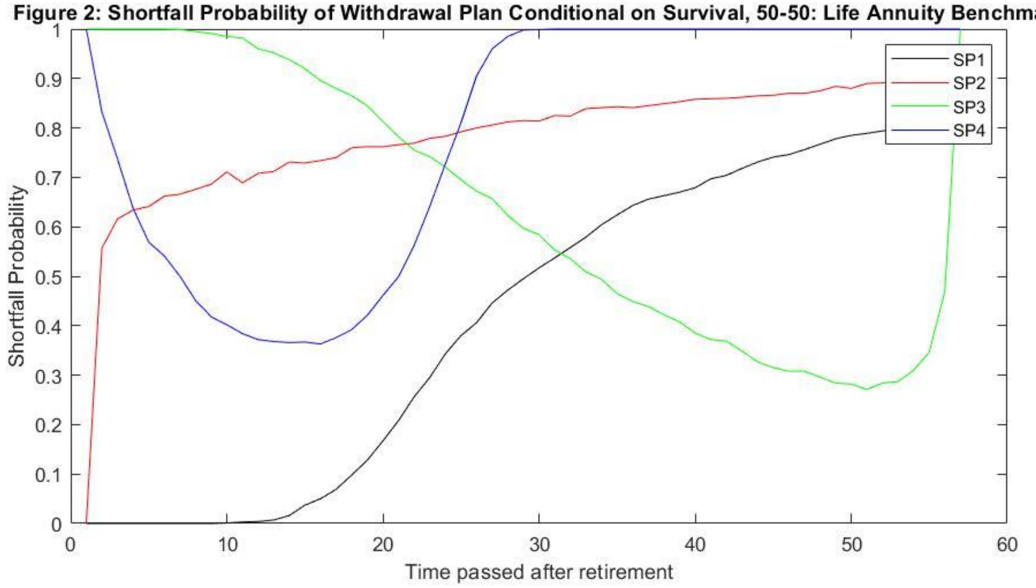

Figure 2 depicts the plots of shortfall probability for the withdrawal strategies. As we can see, the fixed benefit rule is the safest strategy, as it has SP of 0% for the first 13 years, and thereafter has the lowest SP of all (not counting 1/T rule at very late ages). The fixed fraction rule starts on a rather high level of 60% and then increases almost up to 90%. While similar to the result in the original paper in that that it is monotonically increasing, the increase in my case is much bigger: 30% as opposed to only 4 % in the original. This comes, again, from the fact that the withdrawal rate is larger than the average gross interest rate.

Both 1/T and 1/E(T) rules start at high, basically 100% SP, due to very low withdrawal fractions in the early periods. SP for1/T rule gradually decreases over time, reflecting accumulation of assets and the increase in withdrawal fraction. It again rises at the last ages by construction. For1/E(T) rule the SP also decreases over time, but at a higher rate, and starts increasing again after 15 years, hitting 100% at about the age of 95. As mentioned above, the withdrawal fraction increases faster for 1/E(T) rule then for 1/T rule, which does not make enough time for assets to accumulate. Additionally, the remaining lifetime decreases fast during the first 20 years, from 21 to 3, and later starts decreasing very slowly, being close to 1 for the last 20 years. For this reason, the remaining assets are exhausted quickly, and SP reaches 100 again. For this plot also the overall results closely follow those in the original paper.

Figure 3 plots the Shortfall Expectations at each time period, as a percentage of real annuity for comparison. The fixed payment rule looks as the safest, as has the lowest SE up until the age of 100, which is what most people will be concerned about. Increasing SE again reflects decreasing payments due to uncertainty from returns. The fixed fraction rule starts at close to 0% due to its construction, but increases immediately, although at a slower rate than the fixed benefit rule, and has SE even lower than fixed benefit rule after 35 years.

1/T rule starts very high, at about 70%, however, it gradually decreases, equaling the other strategies after 25 years, and thereafter is the least risky strategy (not counting the last years, where the assets are exhausted by construction). This should not come as a surprise, given 1/T rule has increasing benefits and decreasing SP over the years for the reasons mentioned above. Thus, this strategy is the least risky for individuals interested in long term, while is the riskiest for short term oriented retirees. 1/E(T) rule exhibits what the authors of the original paper call an “unexpected behavior”. It starts low, at about 18%, slightly decreases over the next 20 years, but then drastically increases to 100% afterwards. This may be explained by highly increasing withdrawal fractions that quickly exhaust the remaining assets.

Overall, the plots in the figure follow the same patterns as in the original paper, although deteriorate slightly faster. The fact that they end up at higher SEs may be explained by 11 more years that are included in my plots.

Figure 3: Shortfall Expectation of Withdrawal Plan Conditional on Survival, 50-50: Life Annuity Benchmark

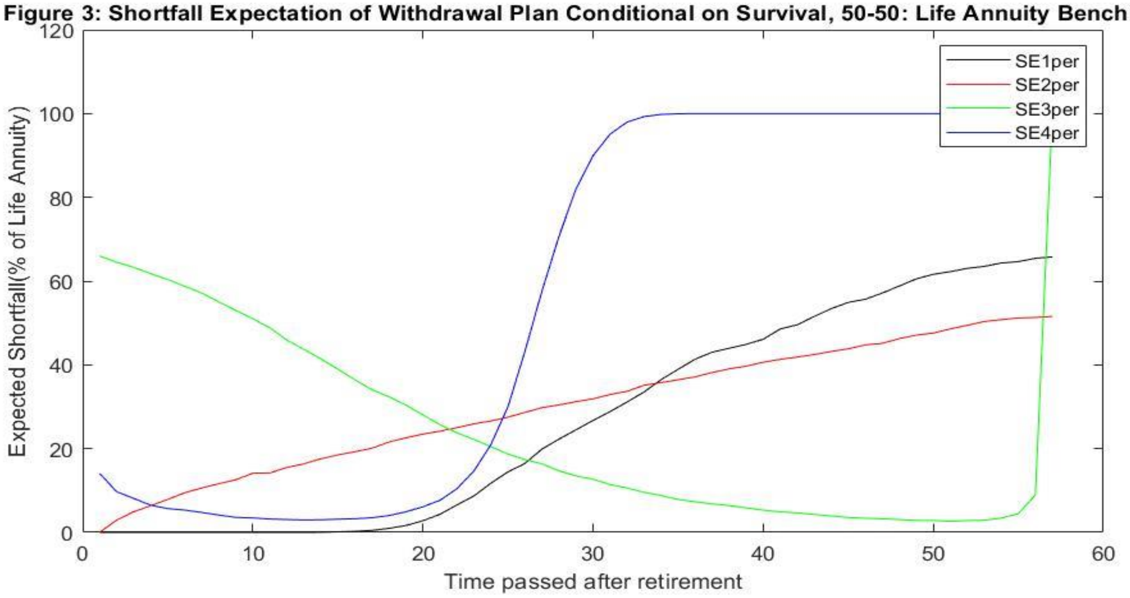

Figure 4 shows the mean bequest of the four strategies as a percentage of initial capital. I took as bequest simply the remaining asset values, and not their present values, which are discounted by the risk free rate and weighted by probabilities. Also, as the initial capital is 100 in my case, the values are already as a percent of it.

The bequest offered by the fixed fraction rule starts at 100, then decline a little, and increase back to almost the initial level at very late levels. I cannot offer a precise explanation for such a behavior, but it is probably caused by stochastic returns. The fixed fraction rule decreases steadily, which is expected, as it was shown that the withdrawal rate is larger than the average gross return. It is also the strategy that shows the least fluctuations in bequest.

For 1/T rule, the hump in the bequest reflects the accumulation of assets in the early periods due to low withdrawal rate. After 35 years it starts declining until it hits 0. Yet, this strategy will be the most attractive for those interested in leaving a bequest, as it offers the highest bequest till 110 years. 1/E(T) offers the lowest bequest of all the strategies and it also declines the fastest, hitting 0 in 30 years. It will be the least desirable choice for a retiree concerned about leaving a bequest to the heirs.

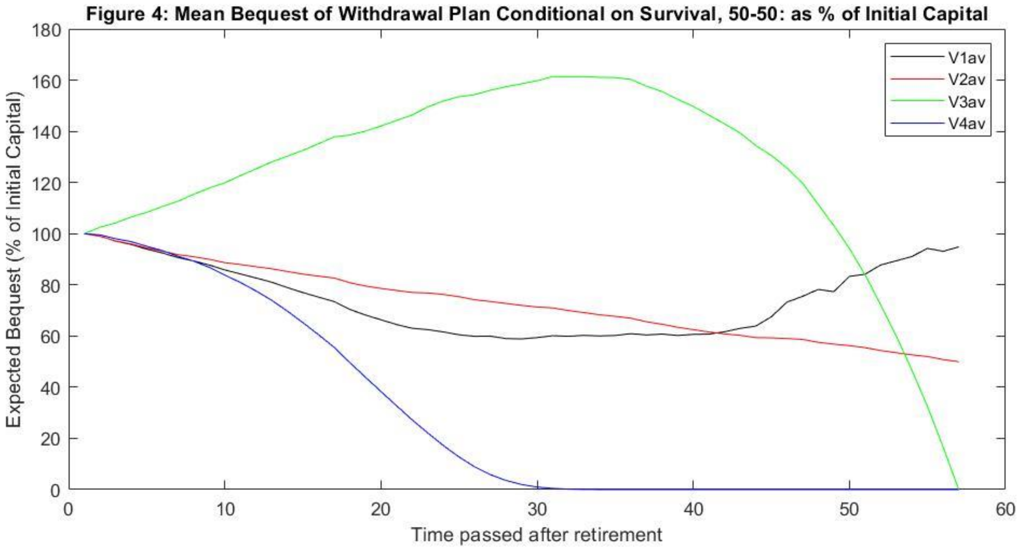

Table 1 shows the results of optimization. It shows the values for EPV Shortfall, EPV Benefits, and EPV Bequest for each of the four strategies, as well as for the life annuity. Additionally, it shows the results for each strategy optimized by minimizing the EPV Shortfall, and the respective asset allocation weights are also shown. The table provides the information for male and female, in both cases for retirement age 65.

Table 1: Sex-specific Results for Risk-Minimizing Phased Withdrawal Strategies

A. Results for Male, age 65: Benchmark real life annuity 5.1583 € p.a./per 100€					
				Investment weights, %	
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Equity	Bonds
1.Real Annuity	0	99.6491	0	0.5	0.5
2. Fixed Benefit	2.2525	94.9734	38.4637	0.05	0.95
3. Fixed Pct.	15.2592	91.1281	71.832	0.65	0.35
4. 1/T	40.3004	80.9791	193.3371	1	0
5. 1/E(T)	11.248	95.2728	27.609	0.05	0.95
B. Results for Female, age 65: Benchmark real life annuity 4.4896 € p.a./per 100€					
6.Real Annuity	0	99.9664	0	0.5	0.5
7. Fixed Benefit	0.7348	97.6154	44.4887	0.05	0.95
8. Fixed Pct.	12.1587	89.4807	64.9483	0.35	0.65
9. 1/T	32.6425	107.2024	210.067	1	0
10. 1/E(T)	9.1038	103.3195	25.5218	0.05	0.95

Rows 1 and 6 show the data for real life annuity. It has EPV of shortfall and bequest of 0, and EPV Benefits almost 100. Although the periodic payment for women is less, their survival probabilities are higher, which explains why the values for both genders do not differ much.

Rows 2 and 7 show the same values for the fixed benefit strategy. We can see that in both cases the expected benefits are lower when compared to life annuity, but the withdrawal strategy introduces shortfall and bequest. Also for both genders the optimal allocation was 5% in stocks and 95% in bonds. From the results of my data the two asset classes had almost the same return but bonds were less volatile. Seemingly this strategy benefits from less volatility of the returns. It is also noteworthy that all the values are better for female than for male: higher benefits and bequest, while lower shortfall. This is likely caused by a smaller benchmark for shortfall, and smaller withdrawals, which allow the assets to accumulate more value in general.

Rows 3 and 8 show the results for fixed percentage rule. For both genders this rule offers much more shortfall, less benefits, but at the same time higher bequest. Interestingly, this is the only strategy for which optimal asset allocation weights for both genders were not the same. Although the differences in shortfall values in case of females under different weights were minuscule. Unlike the previous strategy, this strategy is more favorable for males: with a little less risk it offers higher benefits and bequest. But like the fixed benefit rule, this rule also offers less benefits than the life annuity.

Rows 4 and 9 provide the respective values for the 1/T rule. As can be seen from the table, it is the most volatile strategy, inasmuch as it provides the highest shortfall, but also the highest returns in the form of benefits (for women) and bequest. This strategy will especially appeal to those prioritizing bequest, since the values for expected bequest are very high compared to other strategies and to the invested amount. The optimal asset allocation is somewhat peculiar though, as it consists only from stocks, which are more volatile. This strategy also more favorable for females in all aspects.

Finally, rows 5 and 10 display results for the 1/E(T) strategy. It offers relatively low shortfall, high benefits, but the lowest bequest of all strategies. The optimal asset allocation is again 5% stocks and 95% bonds. The strategy is slightly better for females, although offers less bequest for them.

4. Conclusion

In this paper I have replicated a paper by Dus, Maurer, and Mitchell (2005) with newer data. With some minor deviations from their paper, I followed the methodology of the authors, and got results that are consistent with the original paper.

The paper compares several retirement strategies, including life annuities and four phased withdrawal strategies. Due to the stochastic nature of asset returns, most numbers are calculated using Monte-Carlo simulation for obtaining different estimates of risk and return. The results show that there is no dominant strategy among these in terms of the risk and returns. Each strategy offers a specific risk-return combination, and depending on preferences of a retired person the desirable strategy will change. For example, life annuities eliminate longevity risk, but remove control and bequest potential. Fixed benefit rules are the least risky, whereas fixed fraction rule fluctuates the least. 1/T rule offers the highest amount of bequest and payments in later years, but has high shortfall probabilities and low payments in the early years. While 1/E(T) rule would be preferred by short-term oriented individuals, as it pays the most in the early years, but exhausts fast and leaves the lowest bequest. Yet these strategies create flexibility in decision making of a retiree.

It was also apparent that the strategies are generally more favorable for women rather than men. In almost all scenarios women face lower risk while having higher returns. The difference primarily comes from the lower mortality rates for women, which also leads to a smaller annuity payment benchmark.

References

Albrecht, P. and R. Maurer (2002). "Self-annuitization, Consumption Shortfall in Retirement and Asset Allocation: The Annuity Benchmark". Journal of Pensions Economics & Finance 1(3), pp. 269-288.

Dus, I., R. Maurer, and O. S. Mitchell (2005). "Betting on Death and Capital Markets in Retirement: A Shortfall Risk Analysis of Life Annuities versus Phased Withdrawal Plans". Financial Services Review 14(3), pp. 169-196.